WITHOUT GEOMETRY, LIFE IS POINTLESS

MUSINGS ON MATH, EDUCATION, TEACHING, AND RESEARCH.

My research in a (rather large) nutshell

FRIDAY, SEPTEMBER 3, 2010

Habits of Mind

This is still a work in progress (and feedback would be greatly appreciated), but I've decided to explicitly teach (and assess...more on that later) 4 "categories" of mathematics this year.

1. Skills (I know how to...)
2. Concepts (I understand and can explain why...)
3. Connections (I see and can explain the relationship between...)
4. Mathematical Habits of Mind (I can use and appreciate the process of...)

I've decided not to use the term "problem solving" because I believe this term is often misused to include solving problems and because the motivation for problem solving skills seems to be to help you get an answer. While I believe that they can be very helpful in finding answers, I see mathematical habits of mind as also being mathematical in and of themselves. So...while searching for patterns may help you solve a problem it is also DOING mathematics.

Here's the current version of the mathematical habits of mind I think are important. I hope to explore (in varying depths) every one of these and have already shared the list with my 6th graders.

This is definitely a work in progress and some of these are based on work by Cuoco, Driscoll, Schoenfeld, and others.

Habits of mind

1. **Pattern Sniff**
   A. On the lookout for patterns
   "Ok. We've been working on this staircase problem and it seems that you can't write perfect squares as a sum of consecutive whole numbers."
   B. On the lookout for shortcuts

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Habits of Mind

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ABOUT ME

AVERY

BAY AREA, CA, UNITED STATES

I've been teaching some permutation (or is that combination?) of math and science to third through twelfth graders in private and public schools for 11 years. I'm also pursuing my EdD in education and will be both teaching and conducting research in my classroom next year. Click on the link above my blog posts for more information.

VIEW MY COMPLETE PROFILE

"It would be nice if there were a faster way to do 57x34 than adding 57 to itself 34 times. Think we can find a way?"

2. **Experiment, Guess and Conjecture**
   A. Can begin to work on a problem independently
   "I'm not sure how to solve this problem, but I'm confident I can make some progress."
   B. Estimates
   "Without doing any calculations, I'm guessing that it will take him 30 seconds to walk up the down escalator."
   C. Conjectures
   "Based on my work, I think the following is true."
   D. Healthy skepticism of experimental results
   "Boy, it sure seems like this 4, 2, 1 thing always repeats but we don't have a proof yet."
   E. Determines lower and upper bounds
   "I know it will take the people at least 10 minutes to cross the bridge because the 10 minute soldier has to cross the bridge. I also found a solution that takes 19 minutes so I know the final answer is somewhere between 10 and 19 minutes."
   F. Looks at small or large cases to find and test conjectures
   "I made a table of the first 5 cases and I think I see a pattern. I'm going to see if this pattern holds for the 100^{th} case."
   G. Is thoughtful and purposeful about which case(s) to explore

   H. Keeps all but one variable fixed
   "So I'm exploring the equation y=mx+b and I'm wondering how the graph changes as m and b change. For now, I'm going to set m to 1 and just look at how the graph changes when I change b."
   I. Varies parameters in regular and useful ways (Even/odd example)
   J. Works backwards (guesses at a solution and see if it makes sense)

3. **Organize & Simplify**
   A. Records results in a useful way
   "I'm going to make a table."
   B. Process, solutions and answers are detailed and easy to follow
   C. Looks at information about the problem or solution

FOUNDATIONS OF MY RESEARCH

The Art of Problem Posing

Mathematical Habits of Mind

Mathematical Thinking and Problem Solving

How to Solve It

The Open-Ended Approach: A New Proposal for Teaching Mathematics

A Mathematician's Lament

FOLLOWERS

in different ways

D. Determine whether the problem can be broken up into simpler pieces
"I think I can solve this problem by solving these other 2 simpler problems."

E. Considers the form of data (deciding when, for example, 1+2 is more helpful than 3)
"I'm going to leave my fraction as 6/36 because the 6 represents the number of ways you can roll a 7 with 2 standard dice and the 36 represents the total number of rolls."

F. Uses parity and other methods to simplify and classify cases
"Next time we play 21 Nim I'm going to keep track of whether the running sum is a multiple of 3, one more than a multiple of 3, or 2 more than a multiple of 3."

4. Describe

A. Verbal/visual articulation of thoughts, results, conjectures, arguments, process, proofs, questions, opinions

B. Written articulation of thoughts, results, conjectures, arguments, process, proofs, questions, opinions

C. Can explain both how and why
"The algorithm for dividing fractions is simple. Now I just need to work on making sense why this works."

D. Creates precise problems

E. Invents notation and language when helpful
"For the sugar weighing problem, I don't want to have to write out every solution in words so I'm going to let the symbol 3w~3s stand for the act of putting the 3 pound weight on one side of the balance scale, measuring out 3 pounds of sugar on the other side of the scale, and then setting aside the sugar."

F. Ensures that this invented notation and language is precise
"I need to be careful that I am differentiating between sugar that I am measuring and sugar I am using as a weight."

5. **Tinker & Invent**

A. Creates variations

B. Looks at simpler examples when necessary

C. Looks at more complicated examples when

necessary
D. Creates extensions and generalizations
E. Creates algorithms for doing things
F. Looks at statements that are generally false to see when they are true
G. Creates and alters rules of a game
H. Creates axioms for a mathematical structure
I. Invents new mathematical systems that are innovative, but not arbitrary

6. Visualize

A. Uses pictures to describe and solve problems
B. Uses manipulatives to describe and solve problems
C. Reasons about shapes
"I see how this structure is made."
D. Visualizes data
E. Looks for symmetry
F. Visualizes relationships (using tools such as Venn diagrams and graphs)
G. Vizualizes processes (using tools such as graphic organizers)
H. Visualizes changes
I. Visualizes calculations (such as doing arithmetic mentally)

7. Strategize, Reason and Prove

A. Moves from data driven conjectures to theory based conjectures
B. Tests conjectures using thoughtful cases
C. Proves conjectures using reasoning
D. Looks for mistakes or holes in proofs

8. Connect

A. Articulates how different skills and concepts are related
B. Applies old skills and concepts to new material
C. Describes problems and solutions using multiple representations
D. Finds and exploits similarities between problems (invariants, isomorphisms)

9. Listen & Collaborate

A. Respectful to others when they are talking
B. Asks for clarification when necessary
C. Challenges others in a respectful way when there is disagreement
D. Participates
E. Ensures that everyone else has the chance to participate

F. Willing to ask questions when needed
G. Willing to help others when needed
H. Shares work in an equitable way
I. Gives others the opportunity to have "aha" moments

10. **Contextualize, Reflect and Persevere**
    A. Determines givens
    B. Eliminates unimportant information
    C. Makes and articulates reasonable assumptions
    D. Determines if answer is reasonable
    E. Determines if there are additional or easier explanations
    F. Continuously reflects on process
    G. Works on one problem for greater and greater lengths of time
    H. Spends more and more time stuck without giving up

AT 12:37 PM

4 COMMENTS:

Sue VanHattum said...

I love this! I'm going to ask my students to read this.

SEPTEMBER 3, 2010 1:13 PM

Jason Buell said...

Great list.

I was sent a list from the Park School by John Burke over at quantumprogress and they have:
look for patterns
tinker
guess
describe
visualize
seek proof
use plausibility
take things apart
conjecture
examine a similar problem
use inverse thinking
determine relevance
use multiple points of view
create

Shoot me an email if you want me to send the pdf I have, there are some sample problems and a description attached as well.

Oh and I sent the email to say I'd like to come to the math circle, waiting to hear back. Look forward to meeting you. And Sue if you're in the Peninsula area.

SEPTEMBER 3, 2010 11:17 PM

Dan Meyer said...

Yeah, this is hot.

SEPTEMBER 4, 2010 8:05 AM

Bonita and Family said...

Fun and true list. I believe we should design similar lists in science, social science, reading, writing, and so on...(actually, many of these work for other subjects with a slight change in output and context). These habits of mind are the components that build lifelong learning and need to be better valued in education. I am so glad you are writing about this.

SEPTEMBER 4, 2010 9:45 AM

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